Joint Expansion Planning for Natural Gas and Electric Transmission with Endogenous Market Feedbacks

Russell Bent
Los Alamos National Lab
rbent@lanl.gov

Seth Blumsack
Penn State and Santa Fe Institute
sab51@psu.edu

Pascal van Hentenryck
University of Michigan
pvanhent@umich.edu

Conrado Borraz Sanchez
KPMG
cborrazsanchez@kpmg.com

Scott Backhaus
Los Alamos National Lab
backhaus@lanl.gov

Abstract

The recent and rapid shift towards the increased use of natural gas for power generation has convinced both power grid operators and regulators that additional coordination between electric power and natural gas transmission is needed to ensure the reliable operation of both systems. We report on an ongoing modeling effort for joint gas-grid expansion planning. We develop a Combined Electricity and Gas Expansion (CEGE) planning model that determines least-cost network expansions for power and gas transmission in a way that endogenizes the effects of expansion decisions on locational costs for electric power and natural gas deliveries. The CEGE model, which leverages recent advances in convex approximations for large-scale nonlinear systems, is illustrated on a new gas-grid test system topologically similar to the Northeastern United States. We show that the CEGE model is computationally tractable, and how the model might be used to jointly plan infrastructures to avoid extreme events such as the coincident gas-electric peaks experienced during the 2014 polar vortex.

1. Introduction

As the price of natural gas has declined and other power generation sources such as coal and nuclear face increasing regulatory pressures, power grids in many parts of North America have become increasingly reliant on natural gas as a fuel for the power generation fleet. Despite recent studies suggesting that large power grids could accommodate a substantial fraction of generation coming from natural gas during normal conditions [1, 2], the Polar Vortex incidents during the winters of 2010/11 [3] and 2013/14 [4] represent extreme events that have raised concerns among industry and regulators. Gas-fired power plants without on-site storage or duel-fuel capabilities may face fuel delivery insecurity arising from the interruptible nature of the gas transmission capacity contracts that many generators sign [4]. The asynchronous nature of gas and electric power transmission operations and price formation has also created some uncertainty for gas-fired generators [5].

In this paper, we illustrate a modeling framework, the Combined Electric and Gas Expansion (CEGE) planning model, to address the value of coordination in expansion planning between natural gas and electric power systems. Coordination in planning will become increasingly important as power grids continue to move towards natural gas and plants powered by other fuels retire. The model in the present paper represents an extension of previous work in [6, 7, 8, 9], in which spot price feedbacks from expansion decisions are endogenized and incorporated into the planning objective function. We illustrate this modeling approach using an integrated test system composed of the IEEE 36-bus NPCC electric power system [10] with marginal costs for coal, nuclear, hydro, wind, oil and refuse generation as reported in [11] and a multi-company gas transmission network covering the Pennsylvania-To-Northeast New England area in the US [8]. We implement two types of planning models with our model formulation and test system. The first is an **Expansion-only Model** which seeks the lowest-cost set of network expansions to meet a specific level of demand. The second is a planning model with **Endogenous Price Formation** which seeks to jointly minimize network expansion and operation costs, recognizing that network expansion choices will feed-forward into market price outcomes.

The modeling framework demonstrated in this paper represents a computationally tractable model for the integrated analysis of operational and network expansion decisions for coupled energy infrastructures. Our work adds to a small but growing body of literature that considers joint expansion planning for natural gas and

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electric power transmission [12, 13, 14, 15, 16, 8, 9]. The portion of our model that captures operational decisions for these two infrastructures builds on prior work ([17, 18, 19, 20, 21, 22, 23]) in several distinct ways. First, our modeling framework is computationally tractable but does not impose transportation type flow or other linear approximations, such as those in [17, 12, 13, 14, 16, 19, 20, 21, 22]. Second, our model appears to be the first to incorporate endogenously determined spot gas and electricity prices into a joint gas-grid expansion planning model in a way that permits the joint optimization of capital and operational costs. As mentioned earlier, this is critical to explore situations that arised in the polar vortex events.

The rest of this paper is organized as follows. Section 2 presents the formulation of the CEGE planning problem. Section 3 describes the new test system on which we illustrate the CEGE model. Section 4 discusses the numerical experiments. Finally, Section 5 presents our conclusions and directions for future work.

2. CEGE Optimization Problem

In this section we present the CEGE optimization problem. This problem consists of constraints and variables associated with modeling the non-convex physics of electric power and natural gas systems, modeling expansion options and costs, modeling heat-rate curves, and incorporating power generation costs. For the rest of this paper, bold face is used to indicate constants. All edges in the model are undirected, however, by convention, we assume an arbitrary orientation. Thus, for an edge a, a_{ij} refers to the arbitrary orientation of a from node i to node j. This convention is used when linking a bus i to a. In this case, a_{ij} refers to those edges oriented from i to a node j and a_{ji} refers to those edges oriented from a node j to i.

2.1 Electric Power Model

The AC physics of electric power systems are governed by Kirchoff's and Ohm's laws. We use an AC type power flow model in the CEGE framework to highlight how recent modeling advances have made AC flow approaches more tractable, and because in our simulation experiments we found some binding voltage constraints that would not appear in a DC type of power flow model. Within the CEGE, we use constraints

$$\sum_{j \in G_i} p^g_j - p^l_i - gs_i v_i^2 = \sum_{j \in N^e_i} p_{ij} \quad \forall i \in N^e, \quad (1)$$

$$\sum_{j \in G_i} q^g_j - q^l_i + bs_i v_i^2 = \sum_{j \in N^e_i} q_{ij} \quad \forall i \in N^e, \quad (2)$$

to model Kirchoff's laws. Here, N^e, G_i, and N^e_i model the sets of all buses (nodes), the generators connected to bus i, and the buses connected to bus i by an edge respectively. The variables p_{ij} and q_{ij} model the active and reactive power leaving bus i on an edge to bus j respectively. Similarly, variables p^g_j and q^g_j the active and reactive output of generators. The notation v_i is used to denote the voltage magnitude of bus i. Finally, p^l_i, q^l_i, gs_i, and bs_i are used to denote active load, reactive load, active compensation, and reactive compensation. The lossy Ohm's law is then modeled with these constraints

$$p_{ij} = \frac{g_a}{r_a^2+\Delta_a^2} v_i^2 - \frac{(g_a r_a + b_a \Delta_a)}{r_a^2+\Delta_a^2} v_i v_j \cos(\theta_i - \theta_j) - \frac{b_{ij} r_{ij} - g_{ij}\Delta_{ij}}{r_{ij}^2+\Delta_{ij}^2} v_i v_j \sin(\theta_i - \theta_j), \forall a = a_{ij} \in A^e, \quad (3)$$

$$p_{ji} = g_a v_i^2 - \frac{(g_a r_a - b_a \Delta_a)}{r_a^2+\Delta_a^2} v_i v_j \cos(\theta_j - \theta_i) - \frac{b_a r_a + g_a \Delta_a}{r_a^2+\Delta_a^2} v_i v_j \sin(\theta_j - \theta_i), \forall a = a_{ij} \in A^e, \quad (4)$$

$$q_{ij} = -\frac{b_a + \frac{c_a}{2}}{r_a^2+\Delta_a^2} v_i^2 + \frac{b_a r_a + g_a \Delta_a}{r_a^2+\Delta_a^2} v_i v_j \cos(\theta_i - \theta_j) - \frac{g_a r_a + b_a \Delta_a}{r_a^2+\Delta_a^2} v_i v_j \sin(\theta_i - \theta_j), \forall a = a_{ij} \in A^e, \quad (5)$$

$$q_{ji} = -b_a + \frac{c_a}{2} v_i^2 + \frac{b_a r_a - g_a \Delta_a}{r_a^2+\Delta_a^2} v_i v_j \cos(\theta_j - \theta_i) - \frac{g_a r_a - b_a \Delta_a}{r_a^2+\Delta_a^2} v_i v_j \sin(\theta_j - \theta_i), \forall a = a_{ij} \in A^e, \quad (6)$$

The notation g_a, b_a, and c_a are used to denote the line conductance, suseptance, and charging respectively. Parameters r_a and Δ_a are then used to model the transformer tap ratio and transformer phase shift. These are set to 1 and 0 respectively, for non-transformer lines. The notation θ_i is used to denote the voltage phase angle at bus i. The thermal limits of the lines are modeled using constraints

$$p_{ij}^2 + q_{ij}^2 \leq \zeta_a^2, \quad \forall a = a_{ij} \in A^e, \quad (7)$$

$$p_{ji}^2 + q_{ji}^2 \leq \zeta_a^2, \quad \forall a = a_{ij} \in A^e, \quad (8)$$

where ζ_a is the rating of the line. Finally, we bound voltage magnitudes and generator output with these constraints

$$\underline{p}^g_i \leq p^g_i \leq \overline{p}^g_i, \quad \forall i \in \Omega, \quad (9)$$

$$\underline{q}^g_i \leq q^g_i \leq \overline{q}^g_i, \quad \forall i \in \Omega, \quad (10)$$

$$\underline{v}_i \leq v_i \leq \overline{v}_i, \quad \forall i \in N^e, \quad (11)$$

where Ω denotes the set of all generators. The underline and overline notation is used to denote the lower and upper bounds of these variables.

Expansion variables for new power lines are denoted with z^e_a. These variables are used to set q_{ij} and p_{ij} to 0 and turn off Constraints 3-6 when $z_a^e = 0$. For example, Equation 3 becomes

$$p_{ij} = z_a(\cdot), \; \forall a = a_{ij} \in A^e, \qquad (12)$$

where $(\cdot)$ is used to denote the right-hand side of Equation 3. The full details of this disjunctive model are provided in [24] in the context of line switching. Furthermore, this model of AC power flow physics is not convex and often computationally difficult to solve. To address this complexity, we use the second-order cone relaxation discussed in [24].

2.2 Natural Gas Model

The CEGE model presented here assumes steady-state gas flow and models a single period of gas system operation. Multi-period gas flow simulation, even in steady-state, would necessitate linking equations to represent the slow dynamics of gas flow as compared to power flow. Incorporation of multi-period gas flow into the CEGE framework represents an area of ongoing model improvement. The steady-state physics of natural gas systems are modeled using the Weymouth equations. The Weymouth equations connect the flow of gas to the difference in pressure using the constraint

$$(\pi_i - \pi_j) = w_a |x_a| x_a, \quad \forall a = a_{ij} \in A_p^g, \qquad (13)$$

where A_p^g denotes the set of pipelines in the natural gas system. Here, π_i is used to denote the pressure squared at natural gas junction (node) i, w_a is the resistance factor of the pipe, and x_a is the flow of gas in the pipe. Flow balance at the junctions is preserved using constraints

$$\sum_{a=a_{ij}\in A^g} x_a - \sum_{a=a_{ji}\in A^g} x_a = s_i - d_i - \hat{d}_i, \quad \forall i \in N^g, \quad (14)$$

where A^g denotes all edges in the gas system. The notation s_i, d_i, and $\hat{d}_i$ is used to model gas production, flexible gas consumption, and firm gas consumption respectively. N^g is used to refer to all junctions in the gas network. The change in pressure through compressors and control valves are modeled with constraints

$$\pi_i \underline{\alpha_a} \le \pi_j \le \pi_i \overline{\alpha_a}, \quad \text{if } x_a \ge 0, \forall a = a_{ij} \in A_c^g \cup A_v^g (15)$$
$$\pi_j \underline{\alpha_a} \le \pi_i \le \pi_j \overline{\alpha_a}, \quad \text{if } x_a \le 0, \forall a = a_{ij} \in A_c^g \cup A_v^g (16)$$

where $\underline{\alpha_a}$ and $\overline{\alpha_a}$ are used to denote the lower and upper (de)compression ratios (squared). For compressors (the set A_c^g), these values are typically $\geqq 1$ and for control valves (the set A_v^g) these values are typically ≤ 1. Control valves also include on/off variables to turn off these constraints and set $x = 0$ (see [6]). Finally, flexible consumption, production, and pressures are bound by the following constraints

$$\underline{d}_i \le d_i \le \overline{d}_i, \quad \forall i \in N^g, \qquad (17)$$
$$\underline{s}_i \le s_i \le \overline{s}_i, \quad \forall i \in N^g, \qquad (18)$$
$$\underline{\pi}_i \le \pi_i \le \overline{\pi}_i, \quad \forall i \in N^g, \qquad (19)$$

Once again, the underline and overline notation is used to express the upper and lower bounds of these variables. Expansion variables for new natural gas pipelines are denoted with z_a^g. These variables are used to set x_a to 0 and turn off constraints 20 when $z_a^g = 0$, i.e.,

$$z_a(\pi_i - \pi_j) = w_a x_a^2, \quad \forall a = a_{ij} \in A_p^g, \qquad (20)$$

Furthermore, this model of natural gas physics is not convex and generally computationally intractable to solve. To address this complexity, we use the exact disjunctive formulation to model the flow direction of gas and the subsequent second order cone relaxation as discussed in [6].

2.3 Heat Rate Model

The electric power and natural gas systems are connected by constraints

$$d_i = \sum_{j\in\Gamma_i} (h_1^j + h_2^j p_j^g + h_3^j (p_j^g)^2), \forall i \in N^g, \qquad (21)$$

that express how gas is consumed by electric power generators to produce power. This is often referred to as a heat rate curve. Here h describes the heat rate coefficients of a quadratic curve and Γ_i refers to those generators that consume gas at junction i. In the model, we use $h_3 = 0$, so the constraint is convex. However, when $h_3 \neq 0$, this is a non-convex constraint and equality can be relaxed with $\ge$. This relaxation allows solutions that consume more gas than the generator needs. Since the objective function (discussed later) penalizes congestion, this relaxation is generally tight. However, if it is beneficial to consume more gas, i.e., to lower pressure, then solutions will not be tight.

2.4 Endogenous Gas Price Determination

One of the key contributions of this paper is the endogenous modeling of natural gas prices. Our framework incorporates the modeling of price changes for natural gas that would arise from the construction of new natural gas pipelines, which would decrease congestion in the natural gas transmission system and thus lower prices in constrained areas, or increased demand for natural gas, which would tend to increase prices. Our modeling framework considers the post-expansion gas prices when determining whether a given network expansion option belongs in the cost-minimizing solution. We refer to this modeling approach as *endogenizing natural gas prices* because the prevailing gas price is determined by the chosen set of expansions, which in turn are chosen in part based on their impacts on the natural gas price. The gas pricing model used in this paper permits location-specific natural gas price sensitivities. The price sensitivity of natural gas can be based either on locational demand or on locational pressure.

We illustrate this in Section 4 using a zonal approach instead of a nodal approach where locational prices are calculated using Lagrange multipliers on flow balance constraints (as would be the case with Locational Marginal Pricing on the electrical grid). Gas markets in the U.S. operate as bilateral or over-the-counter markets, so there is no centralized gas system operator clearing the market and calculating associated nodal prices. We represent the gas market in the present paper by building an empirical price sensitivity curve that links historical levels of demand and pressure to historical spot price levels at certain locations on the gas transmission system.

For a pricing zone, $t \in T$, where T is the set of all zones, we calculate the cost of gas, ψ_t, with

$$\psi_t \geq m_1^t + m_2^t \gamma_t + m_3^t \gamma_t^2, \tag{22}$$

where

$$\gamma_t = \sum_{i \in N_t^g} d_i, \tag{23}$$

γ_t is the total amount of flexible gas consumed in t, and N_t^g are the junctions located in t. The coefficients m are used to quadratically price the gas consumption. We calculated the pressure penalty cost based on the maximum pressure in t. The maximum pressure ρ_t is modeled with the constraints

$$\rho_t \geq \pi_i, \forall i \in N_t^g, \tag{24}$$

and the pressure penalty cost ω_t is calculated as

$$\omega_t = n_1^t + n_2^t \rho_t + n_3^t \rho_t^2. \tag{25}$$

The coefficients n are used to quadratically price the pressure. This constraint is also not convex and we relax the equality with $\geq$. Once again, this relaxation is tight because ρ_t only influences a minimization term in the objective function.

For this pricing model we add a minimum price constraint of the form

$$\psi_t \geq C_t \gamma_t, \tag{26}$$

where C_t denotes a linear coefficient on the minimum cost of gas.

The coefficients in the gas pricing model are determined based on historical system and spot price data, as described in Section 3.

2.5 Objective Function

The objective function of the CEGE minimizes the cost of expansion (building pipes and power lines), the cost of gas used by power generators, the cost to produce power for all non-gas fired generators, and the pressure penalty cost, i.e.,

$$\begin{aligned} &\textstyle\sum_{a \in A^e} \kappa_a^e z_a^e + \sum_{a \in A_p^g} \kappa_a^g z_a^g + \sum_{t \in T} \psi_t + \\ &\textstyle\sum_{t \in T} \omega_t + \sum_{i \in \Gamma} \mu_1^i + \mu_2^i p_i^g + \mu_3^i (p_i^g)^2 \end{aligned} \tag{27}$$

where κ^e and κ^g are used to denote the cost of building power lines and pipelines respectively, Γ refers to all generators in the model, and the coefficients μ are used to quadratically cost power production. Since gas is already priced with ψ_t, μ is typically 0 for all gas-fired generators. Also note that $\kappa = 0$ and $z = 1$ for all existing pipes and power lines in the network.

3. Northeastern United States Gas Grid Model

In this section, we describe the joint gas-grid model (NE model) we have constructed to test the CEGE (see Figure 1). This model is representative of the natural gas and electric power systems in the Northeastern United States. While the model has a realistic topology, it was constructed from multiple public sources and does not have the same level of detail as typically featured in electric power system planning cases. The data associated with this model are posted online at `https://github.com/lanl-ansi/micot-gasgrid`.

Electric Power Model The electric transmission system is based on the 36-bus NPCC model first discussed in [10]. Based on the bus names, we geo-located the buses to facilitate coupling the power system to the natural gas system. The original power system model has

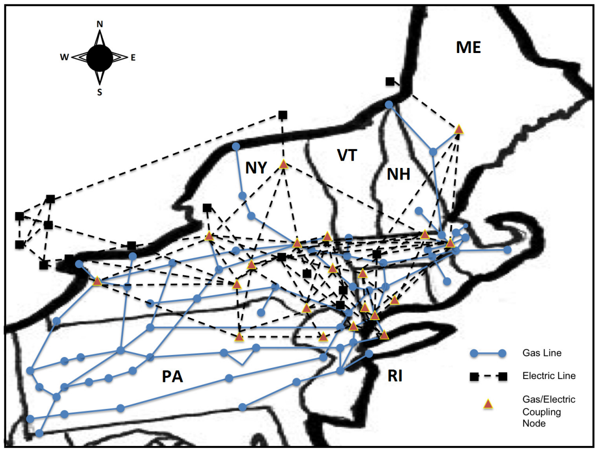

Figure 1: Network structure schematic of the CEGE model for the US New England are: The IEEE 36-bus grid system and the Pennsylvania-to-Northeast gas system.

roughly 10% extra generation capacity. In order to stress electricity consumption in the NE model and focus our studies on the coupling between gas and power (and reflect the current trend to expand generation capacity with gas), we assume that there is infinite extra natural gas generation capacity at existing locations. Thus, the only constraints on satisfying increased demand for power arise from limitations in the natural gas and electric power transmission systems. Future work will include expansion models of generation.

Within the Northeastern United States, the mix of combined versus single cycle natural gas generation is not uniform. For example, the ratio of combined cycle plants to single cycle is much higher in New England than in New York. Locational differences in gas-fired power plant technology mix will naturally affect the costs of both gas and electricity prices. We built such technology variation into our test system by assigning each gas-fired generation node from [10] a share of combined and single cycle gas generation technology. These shares are based on EPA e-GRID data [25] for different utility service territories in the geographic footprint covered by our test system. Within this model, we included power line expansions in parallel with existing lines. Based on [26], the cost of new lines was set to \$1.9M per mile for lines greater than 500 kV, \$1.3M per mile for 345 kV lines, and \$1M per miles for lines of 230 kV or smaller.

Natural Gas Model The natural gas network for the Northeastern United States was constructed by using the gas delivery points described in [9]. We created firm gas demand profiles based on location-specific delivery data on the public posting web sites of natural gas transmission firms operating in the NE geographic area (a complete list of web sites was previously described in [9]). These firm gas demand profiles are assumed to be price-inelastic, and the only price-sensitive demand is assumed to be from electric power plants. Gas source points in our test system were identified by noted marketing points (i.e., points of injection into the gas transmission system) on pipeline atlases published by the gas transmission firms operating in the NE region. We do not include gas storage facilities in our model, although the Leidy field in Northern Pennsylvania is represented in our model as a supply point. Gas storage is an important determinant of regional prices and supply, and will be included in a future version of the test system. The units of the flows are million standard cubic feet per day (Mmscfd). Pipelines between receipt points were built based on information in pipeline atlases posted on the public web sites of natural gas transmission companies. The resistance value of pipes was set using the function described in [27]

$$w_a = c * \frac{D_a^5 (2\log(\frac{3.6 * D_a}{\varepsilon}))^2}{z T L_a \Delta}, \quad \forall a \in A_p^g \tag{28}$$

where c is the gas relative constant (96.074830e-15), D is the diameter of the pipe in mm (we assume all pipes are 762 mm), ε is the absolute rugosity in mm (0.05), z is the gas compressibility factor (0.8), T is the gas temperature in K (281.15), Δ is the density of the gas relative to air (0.6106), and L is the pipe length in km (Euclidean distance). The system has 125 nodes and 143 (existing) edges.

In this model we assume that one pipe can be built in parallel with existing pipes and these parallel pipes have identical characteristics to existing pipes'. The cost of building new pipes was set at \$5M per mile [28].

Gas-Grid model The natural gas generators of the electric power network were linked to the closest natural gas receipt point in the gas system. We used [29] to set the heat rate curve for single cycle gas generators to $h = [0, 0.48, 0]$ and to set the heat rate curve for combined cycle generators to $h = [0, 0.192, 0]$[1]. The model has two price zones based on Transco Zone 6 and the Transco Leidy Zone (extrapolated to include the other gas utilities in their region). We refer to these pricing zones as the cheap gas zone and the expensive gas zone to reflect the modeled difference in price sensitivity to network operating conditions. The zonal pricing models are based on the prices of these areas during January 2014 and were downloaded from the SNL Financial

[1]These numbers are based on converting Mmscfd into BTUs per MW/h

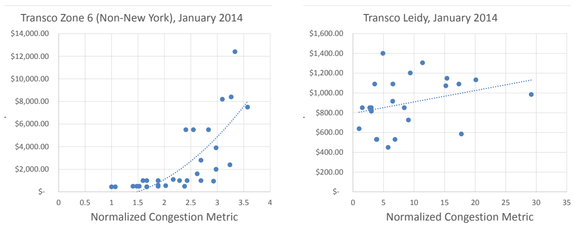

Figure 2: Normalized price sensitivities for the expensive gas zone (left panel, based on Transco Zone 6 Non-New York) and the cheap gas zone (right panel, based on the Transco Leidy zone).

Table 1: Coefficients used in demand and pressure pricing models

Pressure Pricing

Transco Zone 6				Transco Leidy Zone			
Stress	n_1	n_2	n_3	Stress	n_1	n_2	n_3
-	0.0	-0.0064	2e-8	-	794.37	5e-5	0.0

Demand Pricing

Transco Zone 6				Transco Leidy Zone			
Stress	m_1	m_2	m_3	Stress	m_1	m_2	m_3
1.00	0.0	-4641.9	39.436	1.00	0.0	970.77	0.0161
1.10	0.0	-4437.8	40.646	1.10	0.0	963.73	0.0189
1.25	0.0	-4166.8	42.847	1.25	0.0	975.87	0.0129
1.50	0.0	-3714.3	47.446	1.50	0.0	980.97	0.0097
2.00	0.0	-1852.4	54.925	2.00	0.0	991.05	0.0033
2.50	0.0	1447.7	56.546	2.50	0.0	997.13	0.0007
3.00	0.0	5446.3	92,844	3.00	0.0	1001.9	0.0040

database. In Figure 2, we plot the prices as a function of demand, normalized around typical demand, and we fit a convex quadratic polynomial to this data. We then used this curve to create coefficients for price response curves to normalized metrics of congestion (demand and pressure). See Table 1 for the coefficients we used. It is important to note that these coefficients are user inputs and we selected these coefficients for the purposes of exercising the model. We set $C = 450$ for the expensive gas zone and we set $C = 530$ for the cheap gas zone. We also put a minimum price of 0 for the pressure penalty pricing as well. For the expensive gas zone, the price curve is < 0 when the pressure is < 566, so any pressure below this has no penalty. Also, the coefficients for demand-based pricing vary depending on the firm demand in the system. This is because we are only pricing the cost associated with flexible demand.

Network Stress We generated variations of the NE Model to stress the system and analysis the impact of stress to the system design and overall gas prices. We uniformly stressed the power system by multiplying the power demand by a value in $\{1.0, 1.05, 1.10, 1.25, 1.3, 1.35\}$. Similarly, we uniformly stressed the natural gas system by multiplying the firm gas demand by a value in $\{1.0, 1.10, 1.25, 1.50, 2.0, 2.5, 3.0\}$.

4. Numerical Results

In this section, we describe the numerical results of applying the CEGE to the NE Model. We implement two versions of the objective function in the CEGE model. One version finds the lowest-cost expansion plan to meet a given level of demand without considering operational costs. This is referred to as the **Expansion-Only Model** in the results that follow. The second version solves the full CEGE problem including both capital and operational costs. This is referred to as the **Endogenous Price Model** in the results that follow. The convex relaxations of the CEGE model are solved using Gurobi 7.0.1. All computations were performed with an Intel(R) Xeon(R) CPU E5-2660 v3 processor (2.60 GHz) and 62 GB of memory.

We also used the relaxed solutions to attempt to recover feasible solutions to the original non-convex formulation. In the non-convex formulation, we replaced all the binary variables (gas directions, valve status, expansion decisions) with constants based on the assignments of those variables in the relaxed solution. We then solved the full non-convex problem to local optimality using Knitro 9.1.0. We used 1e-4 relative feasibility error for determining feasibility. To improve convergence, we impose constraints that state that the objective function needs to be within 20% of the convex solution.

4.1 Uniform Stress

We illustrate the capabilities of the model by increasing the consumption of gas at each gas demand node by a uniform proportion and increasing the consumption of electricity at each electricity demand node by a uniform proportion. To obtain these solutions, a 12 hour time limit was placed on computing solutions to the MISOCP relaxation of the CEGE. In order to obtain a more reasonable balance between expansion and operational costs, we multiplied the cost of operations by 365 in the CEGE objective function. This represents minimizing the combined expansion cost and operational cost for a full year, assuming uniform daily levels of demand. This assumption can be relaxed in future work to consider peak/off-peak demand scenarios for electricity and natural gas. The pressure penalty cost relative to the cost of gas consumption was adjusted so that these costs had roughly the same order of magnitude within each gas pricing zone. The pressure penalty of Zone 6 is multiplied by 175 and the pressure penalty of the Leidy zone is multiplied by 600.

Table 2 describes the quality of the solutions for different levels of uniform stress for the endogenous-price model. Baseline expansion-only results are also included where the cost of operations are excluded for the objective function. This allows us to isolate design choices made for feasibility from design choices made to improve the operating costs[2]. Except in the most extreme gas stress cases (a 300% increase in gas demand), the CEGE problem is computationally tractable to solve. Generally speaking, the design solutions based on the relaxation have feasible (albeit higher) operating costs. Even in those cases where a feasible solution is not found, the worst infeasibility is relatively small.

The design and operating choices for both methods (the expansion-only model and endogenous-price model considering both expansion and operational costs) are shown in Table 3. In the expansion only model, there are fewer expansions in both the natural gas and electric power transmission systems built for feasibility requirements (i.e., for reliability). Once the power system is stressed by 25% in the expansion-only model, one power line is built. Even at 35%, only five power lines are built. Similarly, gas pipelines are only built in the 300% stress case. In contrast, the endogenous-price model encourages additional expansions to decrease operational costs. In the gas stress cases $< 300\%$, between $5 - 10$ additional power lines are added to the network. These choices are made to shift gas demand used to produce power from the high-cost gas zone in the eastern portion of our test system to the low-cost gas zone in the western portion of our test system. In this case, gas by wire is the cheaper option to deliver additional electricity to the constrained area of our network. This is in some contrast to prior work [30], which suggests that in the presence of static prices transportation of gas via pipeline is more cost-effective than moving gas by wire. The operating condition results for the gas stress 300% are even more interesting. Here, the maximum pressure in the high gas price zone for the expansion-only model is near the upper limit and incurs a very high penalty pressure cost[3]. In contrast, the endogenous-price model builds significantly more gas pipelines to drive those pressures down. These observations are reinforced by Table 4 where the actual operating costs are shown. Here, the extra pipes and power lines are clearly used to drive the costs associated with Zone 6 down.

[2]Objective values are not directly comparable as the operating costs for the expansion only solutions do not include the actual operating costs

[3]As a post processing step, we minimized the operating costs for the expansion design to produce these results.

4.2 Pressure Penalty Budget Constraints

We also considered expansion problems that avoid extreme situations like the polar vortex event during the winter of 2014. We impose a limit on the pressure penalty costs and determine how best to expand the network while staying within the prescribed limit. The results are described in Table 5, where the limit is computed using the penalty obtained in the optimal solution of the CEGE problem presented earlier. In particular, the results present the network expansion when the limit is set to 100%, 10%, 5%, 1%, and 0% of the penalty for the 300% GS and 35% PS problem. Interestingly, the gas network must be expanded by another 40% compared to earlier results to avoid the types of price spikes that were observed during the polar vortex, as soon as the penalty is limited to 10% or less. The electricity network is not affected in this case, consistent with the observation that gas transmission was scarce during the polar vortex, not gas supply *per se*.

5. Conclusion

We have developed and demonsrated a computationally tractable framework for modeling expansion planning decisions in nonlinear natural gas and electric power transmission systems (Combined Gas-Electric Expansion, or CEGE) that can identify cost-minimizing network expansions made for reliability reasons and expansions made for economic reasons. Our modeling framework also uses a data-driven approach to endogenizing the impacts of network expansion on natural gas and electric power operational costs.

The modeling framework has been illustrated on a new joint gas-grid test system under varying demand scenarios for natural gas and electric power. Our simulation results suggest that when demand increases by moderate amounts (25% or 30% in the electric power grid, for example) the natural gas cost impact is minimal and any network expansions can be attributed to the need to maintain sufficient delivery capacity to high-demand areas. At higher levels of demand growth, however, a mix of reliability-driven and economic investments emerge from our modeling framework. We also find that the decisions to build natural gas or electric power infrastructure to serve higher electricity demand are substitutable, and whether it is cheaper to move larger quantities of gas to local power stations or to move larger quantities of electricity over long distances varies by location and natural gas price sensitivity.

The primary contributions of the present paper are

Table 2: The CEGE solutions for the uniformly stressed NE Model. The columns are used to stress the power system (PS). The rows are used to stress the gas system (GS). All objective values are scaled by 1.0E8 and all optimality gaps are expressed in terms of %, except when a feasible solution is not found. Here, we report relative feasibility error with no %. $\underline{\text{Obj}}$ and $\underline{\text{Gap}}$ report the objective value of the relaxed solution and its gap, respectively. $\underline{\text{CPU}}$ reports the cpu time, in seconds, to find the relaxed solution. TO denotes time out (12 hours). $\overline{\text{Gap}}$ is the gap between the primal feasible solution and the relaxation lower bound (or relative feasibility error). $\overline{\text{Obj}}$ is the objective value of the primal feasible solution (or least infeasible primal solution).

Expansion-Only Model

	1.0 PS					1.1 PS					1.25 PS					1.3 PS					1.35 PS				
	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$
1.0 GS	0	0%	10	0	1e-3	0	0%	13	0	2e-3	0.58	0%	4	0.58	1e-4	0.58	0%	15	0.58	7e-4	7.82	0%	21	7.82	0%
1.5 GS	0	0%	58	0	1e-2	0	0%	31	0	3e-3	0.58	0%	9	0.58	0%	0.58	0%	25	0.58	3e-4	7.82	0%	30	7.82	3e-4
2.0 GS	0	0%	214	0	3e-3	0	0%	200	0	2e-3	0.58	0%	36	0.58	6e-4	0.58	0%	24	0.58	4e-3	7.82	0%	23	7.82	2e-3
2.5 GS	0	0%	1950	0	7e-3	0	0%	11790	0	6e-3	0.58	0%	156	0.58	5e-4	0.58	0%	67	0.58	3e-3	7.82	0%	56	7.82	2e-4
3.0 GS	2.74	100%	TO	2.74	3e-4	1.75	100%	TO	1.75	3e-4	2.81	79.2%	TO	2.81	2e-4	2.72	78%	TO	2.72	2e-4	10.6	23.7%	TO	10.6	5e-4

Endogenous-Price Model

	1.0 PS					1.1 PS					1.25 PS					1.3 PS					1.35 PS				
	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$	$\underline{\text{Obj}}$	$\underline{\text{Gap}}$	$\underline{\text{CPU}}$	$\overline{\text{Obj}}$	$\overline{\text{Gap}}$
1.0 GS	40.3	0%	39	48.3	19.9%	49.3	0%	33	59.1	19.9%	63.7	0%	53	76.4	19.9%	70.2	0%	49	84.2	19.9%	82.2	0%	2908	82.2	0%
1.5 GS	41.9	0%	69	50.2	2.1%	51.3	0%	40	61.5	19.9%	66.3	0%	36	79.6	20.0%	72.9	0%	84	87.5	20.0%	85.4	0%	585	102	19.4%
2.0 GS	43.9	0%	484	43.9	0%	53.7	0%	640	64.4	20.2%	69.6	0%	443	69.6	0%	76.4	0%	368	91.7	20.0%	89.1	0%	1438	89.1	0%
2.5 GS	46.2	0.2%	TO	55.4	20.2%	56.4	0.3%	TO	67.8	20.6%	73.1	0.7%	TO	87.7	20.8%	80.2	0.5%	TO	96.2	20.6%	93.1	0.5%	TO	112	20.1%
3.0 GS	60.5	13.1%	TO	59.1	5e-2	73.2	11.8%	TO	87.8	36.1%	95.1	11.8%	TO	114	36%	100.8	9.2%	TO	100.8	9.2%	115	8.9%	TO	115	8.9%

Table 3: Design and operating properties of CEGE solutions for the uniformly stressed NE Model. In this table, z^e refers to the number of power lines that were built; z^g refers to the number of pipe lines built; γ_6 and γ_L refer to the Mmscfd used in Zone 6 and Leidy Zone to produce power; ρ_6 and ρ_L refer to maximum pressure squared in the two two zones.

Expansion Model

	1.0 PS						1.1 PS						1.25 PS						1.3 PS						1.35 PS					
	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$
1.0 GS	0	0	209	567	560	504	0	0	239	643	566	505	1	0	293	745	574	620	1	0	316	779	566	506	5	0	332	822	527	643
1.5 GS	0	0	209	567	531	506	0	0	239	643	566	507	1	0	290	745	619	640	1	0	302	779	823	785	5	0	334	822	593	800
2.0 GS	0	0	209	568	566	551	0	0	239	643	566	549	1	0	287	745	616	820	1	0	301	779	582	1032	5	0	317	822	621	957
2.5 GS	0	0	209	570	563	1153	0	0	239	646	642	1103	1	0	287	751	726	1116	1	0	298	795	903	1096	5	0	273	776	846	1117
3.0 GS	0	0	120	298	995	1159	0	5	160	370	1149	1200	1	5	174	548	1148	1200	1	6	210	437	1141	1200	5	7	285	517	995	1195

Elasticity Model

	1.0 PS						1.1 PS						1.25 PS						1.3 PS						1.35 PS					
	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$	z^e	z^g	γ_6	γ_L	$\sqrt{\rho_6}$	$\sqrt{\rho_L}$
1.0 GS	5	0	181	600	528	503	5	0	211	677	520	504	7	0	253	802	566	508	8	0	269	848	566	506	13	0	274	897	537	507
1.5 GS	5	0	181	600	558	535	8	0	196	693	566	558	8	0	247	810	566	535	10	0	253	865	566	515	13	0	268	897	565	508
2.0 GS	8	0	166	616	566	538	8	0	196	693	566	602	10	0	235	826	566	537	10	0	250	865	566	559	13	0	260	897	565	542
2.5 GS	7	0	162	616	573	1120	8	0	196	694	580	1145	10	0	235	827	606	1144	10	0	249	865	615	1144	13	0	260	897	623	1120
3.0 GS	8	14	158	621	736	1116	9	13	185	696	804	1134	11	14	225	831	845	1125	13	13	234	870	802	1080	16	12	249	902	842	1118

Table 4: Cost properties of CEGE solutions for the uniformly stressed NE Model. Here $\zeta_i = \psi_i + \omega_i$ refers to the total daily amount scaled by 10^6 on procuring gas for generation in each price zone, $\frac{\zeta}{\gamma}$ tracks the daily price per Mmscfd scaled by 10^4.

Expansion-Only Model

	1.0 PS				1.1 PS				1.25 PS				1.3 PS				1.35 PS			
	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$
1.0 GS	0.7	1.0	0.4	0.2	1.1	1.1	0.5	0.2	2.0	1.2	0.7	0.2	2.5	1.3	0.8	0.2	2.8	1.3	0.8	0.2
1.5 GS	1.3	1.0	0.6	0.2	1.8	1.1	0.8	0.2	3.0	1.2	1.0	0.2	4.0	1.3	1.3	0.2	4.1	1.3	1.2	0.2
2.0 GS	2.0	1.0	1.0	0.2	2.7	1.1	1.1	0.2	4.1	1.2	1.4	0.2	4.5	1.3	1.5	0.2	5.1	1.3	1.6	0.2
2.5 GS	2.8	1.1	1.3	0.2	3.7	1.2	1.6	0.2	5.4	1.3	1.9	0.2	6.9	1.3	2.3	0.2	5.6	1.3	2.1	0.2
3.0 GS	4.3	0.8	3.6	0.3	7.9	0.9	4.9	0.2	8.3	1.1	4.8	0.2	9.7	1.0	4.6	0.2	13	1.3	4.6	0.2

Endogenous-Price Model

	1.0 PS				1.1 PS				1.25 PS				1.3 PS				1.35 PS			
	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$	ζ_6	ζ_L	$\frac{\zeta_6}{\gamma_6}$	$\frac{\zeta_L}{\gamma_L}$
1.0 GS	0.4	1.1	0.3	0.2	0.8	1.2	0.4	0.2	1.3	1.3	0.5	0.2	1.6	1.3	0.6	0.2	1.7	1.4	0.6	0.2
1.5 GS	0.9	1.1	0.5	0.2	1.1	1.2	0.6	0.2	2.0	1.3	0.8	0.2	2.1	1.3	0.8	0.2	2.4	1.4	0.9	0.2
2.0 GS	1.2	1.1	0.7	0.2	1.7	1.2	0.9	0.2	2.6	1.3	1.1	0.2	3.0	1.4	1.2	0.2	3.2	1.4	1.3	0.2
2.5 GS	1.7	1.1	1.1	0.2	2.4	1.2	1.3	0.2	3.5	1.3	1.5	0.2	4.0	1.4	1.6	0.2	4.3	1.4	1.7	0.2
3.0 GS	3.7	1.1	2.3	0.2	5.0	1.2	2.7	0.2	7.0	1.4	3.1	0.2	7.1	1.4	3.0	0.2	8.1	1.4	3.3	0.2

Table 5: Properties of the 300% GS, 35% PS problem with the pressure penalty cost capped at a percentage of the penalty cost in the optimal solution. The columns z_e and z_p show the number of power lines and pipes that are built. The last two columns show the objective value scaled by 10^8 and the optimality gap.

Penalty cost cap	z_e	z_p	Obj	Gap
100%	16	12	115	8.9%
10%	14	17	118	11.1%
5%	16	17	119	11.2%
1%	16	17	122	12.4%
0%	16	19	122	11.7%

the development and illustration of a computable CEGE model with endogenous commodity price formation, and the introduction of a new gas-grid test system that serves as a platform for computational CEGE experiments. In the present paper we have chosen to provide detailed results for a limited set of experiments to provide information on the computational performance of our CEGE model as well as to articulate broad insights from a type of planning scenario for which our modeling framework may be particularly well suited.

Future work for the CEGE modeling effort involves investigating demand scenarios where load growth is non-uniform over space; incorporating gas generation expansion scenarios into the CEGE planning models (such as some types of scenarios outlined in [2] for the PJM power grid footprint); and conducting security-constrained joint planning for natural gas and electric power systems.

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